CDF Jet Reconstruction Algorithms and the Underlying Event

Frederick K. Balagaddé
Manchester College
604 E. Coll. Ave., N. Manchester IN 46962

Project Supervisor: Dr. Jay R. Dittmann
Fermi National Accelerator Laboratory*
Batavia, IL 60510
(August 10, 2001)

Abstract

In hadron-hadron collider experiments like the one at the Collider Detector at Fermilab (CDF), a quark (or antiquark) in its final form manifests itself as one or more calorimeter jets, which appear as energy deposits shared among several detector calorimeter towers. The CDF collaboration employs jet-clustering algorithms such as the K_TCLUS and JETCLU to recognize, reconstruct and characterize each calorimeter jet in a meaningful way. In this investigation, we explore the sensitivity of each (K_TCLUS and JETCLU) algorithm to the underlying event by tweaking the calorimeter tower energy (simulating additional underlying event activity) before jet reconstruction of CDF data in proton-antiproton collisions at a center of mass energy of 1.8 TeV. Each algorithm will be judged on the basis of the impurity (fraction of the jet transverse energy, E_T, arising from the underlying event) of the jets it reconstructs at three different size parameters (R = 0.4, R = 0.7 and R = 1.0). Our model favored the K_TCLUS algorithm over the JETCLU algorithm as far as the purity of the reconstructed jets is concerned. Smaller cone sizes resulted in purer leading jets. It was also ascertained that activity due to underlying event results in a systematic upward shift of the jet cross-section. It may also change the position of a jet in $\eta \times \varphi$ space and reshuffle the E_T rankings among the jets in an event.

* Operated by the Universities Research Association, under contract with the U.S. Department of Energy

I. INTRODUCTION

The constituents of the proton - quarks and gluons - are ten thousand times smaller than the proton. In order to study the structure of the proton, a method based on the principle of the "scattering microscope"[1] is used. According to this method, two particles under investigation are smashed together, probing each other as deeply as possible. Detailed analysis of the debris allows the experimenter to deduce a picture of the underlying internal structure of the collision participants.

At the Fermilab Tevatron, protons and antiprotons are accelerated to extremely high energies and made to collide head-on. Each hard-collision converts beam particle energy into dozens of out-going particles. By placing a detector (such as the Collider Detector at Fermilab) around the interaction point, one can measure the properties of all the particles emerging from the collision. A detailed study of their properties gives a better understanding of the proton structure. Due to the way quarks and gluons are bound inside the protons, their scattering at large angles results in the appearance of two (or more) highly energetic, collimated sprays of particles called "particle jets," which show as energy deposits shared among several calorimeter towers (geometric cell units) of the detector, "calorimeter jets". Examination of these jets (the direct manifestations of quarks and antiquarks) and their cross-sections provides invaluable information about the underlying quark-gluon interactions.

The CDF collaboration employs jet reconstruction algorithms such as the K_TCLUS and JETCLU to identify, reconstruct and characterize the calorimeter jets in a meaningful way using the energy and geometry information of each tower. The reconstructed calorimeter jets generally contain extra energy due to additional hadronic products arising from the "spectator partons" (the underlying event), and multiple interactions. Consequently, the measured jets may be significantly more energetic than those intended by nature. In order for jet study to be meaningful, such additional energy deposits must be ascertained and removed. The fraction of the jet transverse energy, E_T, arising from the underlying event is referred to as the "impurity" of the jet.

In judging the merits and integrity of the various jet algorithms, one should consider the "impurity" of the leading jets, since the goal is to find the transverse energy, E_T, which arises solely from the ejected hard-scattered quarks (or antiquarks). Although the energy contribution to jets from the underlying event can be corrected for, it would seem reasonable to choose a jet algorithm that minimizes jet "impurity."

[1] The "scattering microscope" method was first demonstrated Ernest Rutherford

In this paper, we shall explore the sensitivity of each algorithm to the underlying event in proton-antiproton collisions at a center of mass energy of 1.8 TeV. To do this, we shall employ a powerful technique of tweaking the calorimeter tower energy (simulating additional underlying event activity) before jet reconstruction takes place. Each reconstruction algorithm will be judged on the basis of the impurity (fraction of the jet transverse energy, E_T, arising from the underlying event) of the jets it reconstructs at three different size parameters (R = 0.4, R = 0.7 and R = 1.0).

During the course of this project, we developed a module that can be used to modify the tower information of each event before the reconstruction algorithms do their work. This allows for the simulation of the underlying event activity (on Monte Carlo or CDF detector data) at the tower level before jet reconstruction. Better thought-out ways of tweaking the tower information prior to jet reconstruction in the future will give scientists invaluable insight into the behavior of the underlying event and the purity of the jets reconstructed.

II. THEORY

A. Quantum Chromodynamics

According to the Standard Model, there exist six fundamental quarks and leptons as listed in the table 1. All matter is composed of a combination of these particles and their antimatter twins. For example, a proton is formed by a bound state of two up quarks and a down (*uud*), and a neutron is composed of two down quarks and an up (*udd*). Quarks primarily interact via the strong force[2]. They possess fractional charge categorized in three flavors labeled "color". Each quark possesses a color charge of red, green, blue or a corresponding "anticolor" for an antiquark.

Quantum Chromodynamics (QCD) is the theory of strong interactions between quarks mediated by gluons[3]. According to QCD, quarks are subject to the "principle of confinement," which states that, *"the net color charge of all macroscopically observable particles must be zero."* A proton must therefore contain a red, blue, and green quark, resulting in a net color charge of zero: [red] + [blue] + [green] = [white]. Needless to say, solitary quarks have never been observed since they each carry a single quantum of color. The confinement principle may be expressed mathematically in the value of the strong coupling parameter a_s, by the variance of its strength with distance. At very short distances

[2] The strong force is the strongest of the four fundamental forces of nature.
[3] Gluons are the strong force mediators or carriers

or very large energies, the value of a_s remains small, allowing the quarks within the hadrons (protons, antiprotons and neutrons) to rattle around nearly freely.

	Fundamental Particle	Symbol	Charge
Quarks	Up	u	$^2/_3$
	Down	d	$^{-1}/_3$
	Charm	c	$^2/_3$
	Strange	s	$^{-1}/_3$
	Top	t	$^2/_3$
	Bottom	b	$^{-1}/_3$
Leptons	Electron	e^-	-1
	Electron neutrino	ν_e	0
	Muon	μ^-	-1
	Muon neutrino	ν_μ	0
	Tau	τ^-	-1
	Tau neutrino	ν_τ	0

Table 1: The fundamental constituents of matter in the Standard Model.

This unique feature of QCD is referred to as *"asymptotic freedom"* for quarks: at high enough energies, the coupling to the surrounding quarks and gluons may be neglected. As the distance between the quarks increases, the coupling strength increases quickly, causing the potential energy between them to rise rapidly, which confines quarks within a particle of radius ~10^{-15}m.

B. The Collider Detector at Fermilab – CDF

The Collider Detector at Fermilab (CDF), located at one of six nominal interaction regions of the Tevatron, is a large, multipurpose apparatus designed to study proton-antiproton collisions with a center-of-mass energy of up to 2.0 (TeV). The CDF detector is forward-backward and azimuthally symmetric, with a geometric center located at the nominal interaction point. An isometric view of the CDF detector is shown in Figure 1. It measures approximately 27m from end-to-end, extends about 10m high, and weighs over 500 tons. Figure 2 shows a longitudinal planar view of one quadrant of the detector. It is composed of several components: the central tracking system, the calorimeter, and the muon spectrometer. The CDF calorimeter system consists of electromagnetic (EM) and hadronic (HA or HAD) elements that are separated into three main detector regions according to their pseudorapidity coverage (*cf* Appendix for CDF coordinate system).

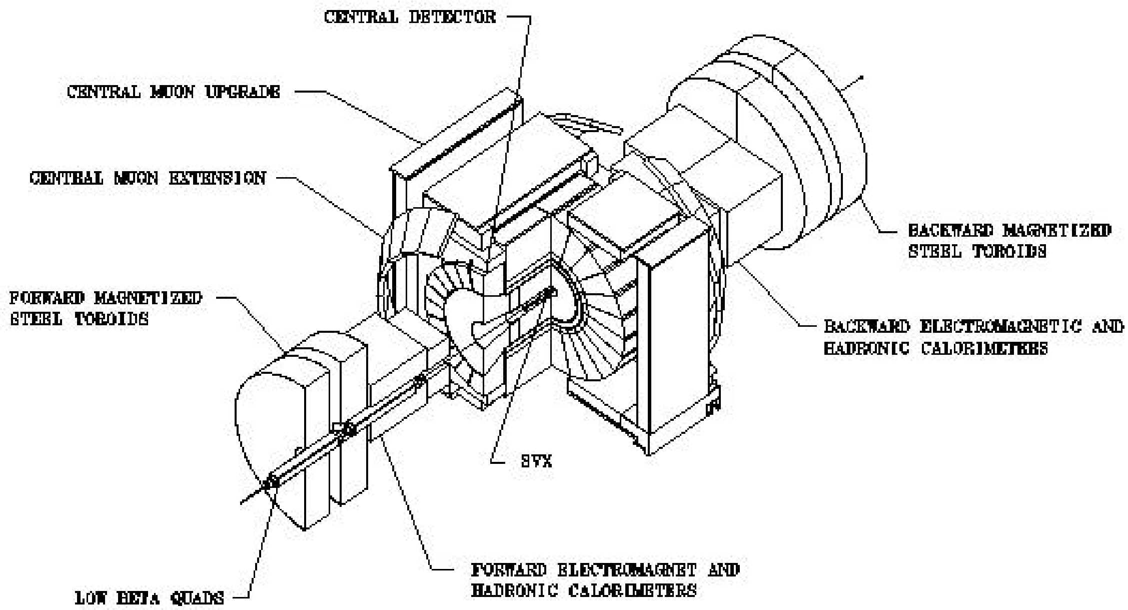

Figure 1: Isometric view of the CDF detector for Run 2.

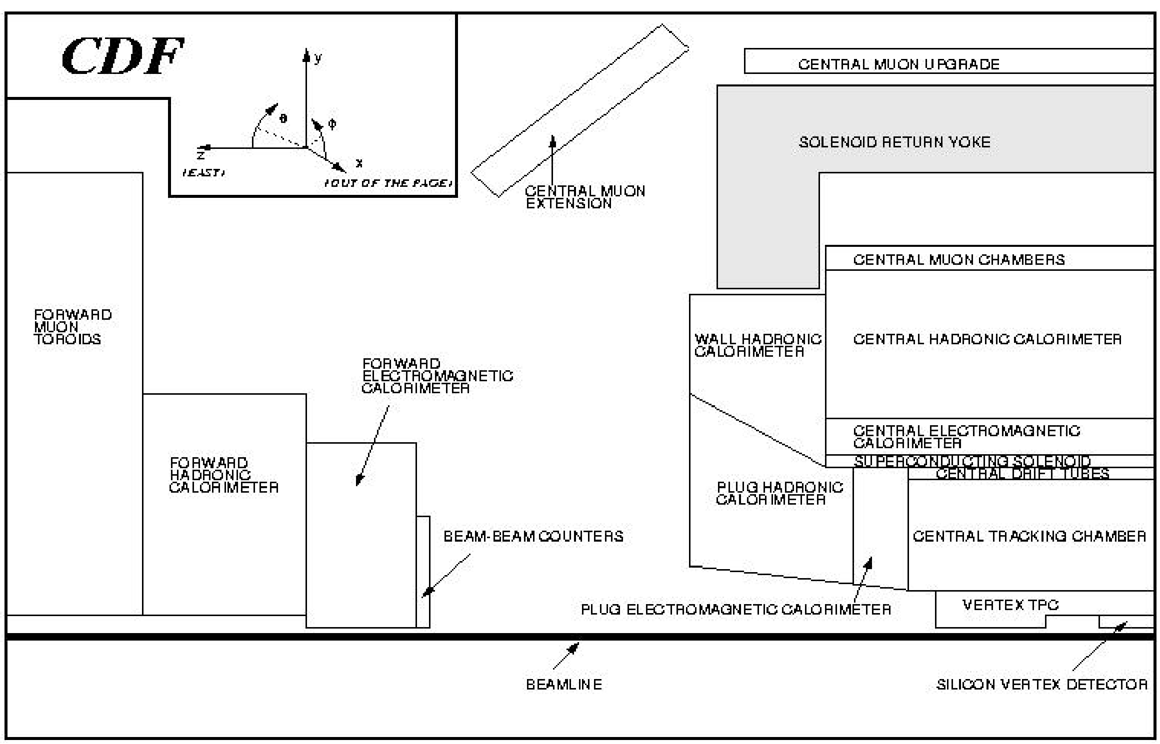

Figure 2: A longitudinal planar view of one quadrant of the CDF detector.

The central region contains the Central Hadron calorimeter (CHA), and the Wall Hadron calorimeter (WHA). The endplug regions contain the Plug Electromagnetic (PEM) and the Plug Hadronic (PHA) calorimeters. The forward (and backward) regions contain the Forward Electromagnetic (FEM) and the Forward Hadronic (FHA) calorimeters. Embedded within the CEM are the Central Electromagnetic Strip chambers (CES), which measure the position of electromagnetic showers as they develop in the calorimeter.

The CDF sampling calorimeters that surround the tracking chambers and solenoid (see Figure 2) are the primary tool for jet energy measurement. This is achieved by totally absorbing the energy of the incoming particle. Upon entering the dense calorimeter medium (lead for the EM or steel for the HAD), hadronic particles initiate particle cascades or showers of particles caused by secondary interactions along the path of the incident particle. The energy is deposited in units known as calorimeter cells. The cell centroids lie along rays of constant pseudorapidity (η) (*cf* Appendix for definitions) drawn from the geometric center of the CDF detector. The cells ganged along the rays of constant η form the CDF calorimeter "towers" of $\Delta\eta \times \Delta\varphi$ transverse segmentation of 0.1×0.1 radians, providing excellent shower position resolution. Each calorimeter tower contains information concerning its geometric location in $\eta \times \varphi$ space and the amount of energy deposited in it.

C. Jets & Jet Production

When hadrons (protons and antiprotons) are accelerated to sufficiently high energies, their constituent partons (quarks and gluons) behave nearly as free independent particles due to asymptotic freedom. Therefore, adequately energetic hadrons can be considered as a broadened beam of loosely bound partons.

At Fermilab, protons and antiprotons are counter-rotated in a super-conducting ring (the Tevatron) 1km in radius and then collided head-on with a center-of–mass collision energy of up to 2.0 TeV (Tera or Trillion electron Volts). Typically, when a proton collides with an antiproton, only two partons, one from each colliding hadron, undergo "*hard-scattering*". The remnants of the colliding hadrons, "spectator partons", do not undergo hard-scattering and the activity due to their interaction is referred to as the "*underlying event*".

Hadronization:

A successful, sufficiently energetic proton-antiproton smash results in a head-on collision between a quark (from the proton) and an antiquark (from the antiproton). The colliding quark and antiquark get dislodged from their parent hadrons and try to escape into isolation. As the separation between the ejected (hard-scattered) parton and its parent hadron increases, the potential energy of the binding (color) force also increases, preventing the parton from escaping into an isolated, colored state. At a critical point as the separation grows, the coupling potential energy stored in the color field tubes manifests itself by spontaneously emitting gluons, which split into quark-antiquark pairs that subsequently recombine into stable, colorless groupings, giving rise to a cascade (shower) of elementary particles (hadrons). This process is known as the "dressing of the quarks", fragmentation, or hadronzation.

Jets:

All the partons in the shower, as well as their products of stable, color-neutral particles gain a boost[4] in the direction of the original partons. In their final state, the partons from the hard-scattering process appear in the form of highly collimated sprays of color-neutral particles (*particle or hadronic jets*), as predicted by QCD. By definition, *a hadronic jet is a shower of particles emitted close to each other in angle during the hard-scattering process.* Hadronic jet production is the dominant process during hadron-hadron collisions with center-of-mass energies greater than ~10 GeV. The parent parton is also usually referred to as a parton jet. As the constituent hadrons of the particle jets pass through the tracking volume and into the calorimeters, they deposit energy in the electromagnetic and hadronic cells, forming *calorimeter jets* (see Figure 3 for a graphical description of the three kinds of jets). A jet-clustering algorithm is then applied to the calorimeter tower data to identify and characterize the calorimeter jets, whose properties epitomize those of the original partons. This gives insight into the properties of the original partons (i.e. momentum & energy). After hadronization, each parton jet manifests itself in the form of one or more calorimeter jets.

[4] "Boost" indicates that the rest frame of the collision is not identical to the laboratory frame

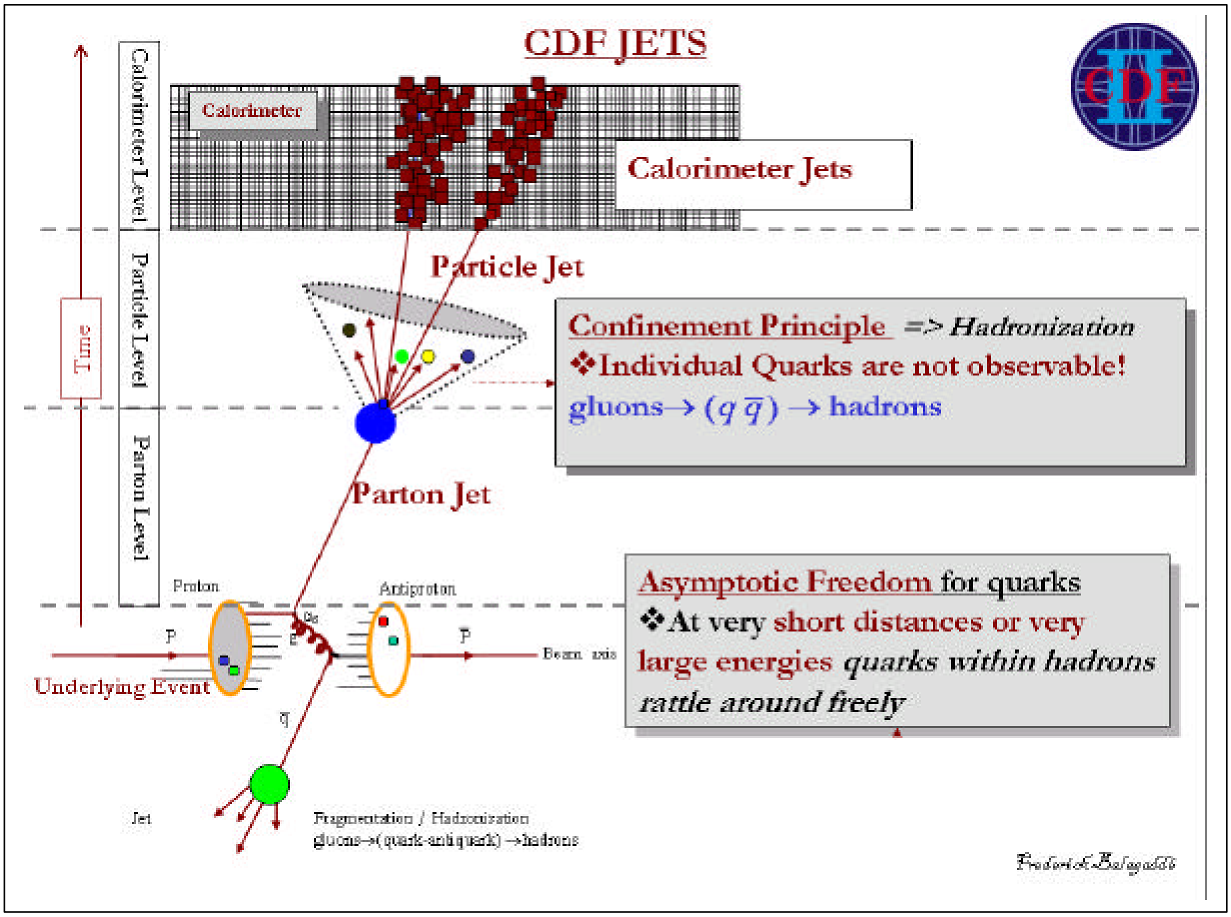

Figure 3: A cartoonist's view of jet production (Not drawn to scale)

The Underlying Event & Jet Impurity:

A typical hard-scattering proton-antiproton collision event consists of outgoing hadrons that originate from the large transverse momentum, dislodged partons and also the beam remnants that originate from the breaking up of the parent proton and antiproton. After a hard interaction, the parent hadrons lose the color charge associated with the ejected partons; and thus their colorlessness and stability. Consequently, in obedience to the confinement principle, the remnants of the parent hadrons (spectator partons) also undergo hadronization. The additional hadronic products arising from the "spectator partons" are collectively called the underlying event. Needless to say, due to the underlying event, the jets measured may be significantly more energetic than the jets intended by nature. In order for jet study to be meaningful, the additional energy deposits due to the underlying event must be determined and removed. The fraction of jet energy arising from the underlying event is referred to as the "impurity" of the jet.

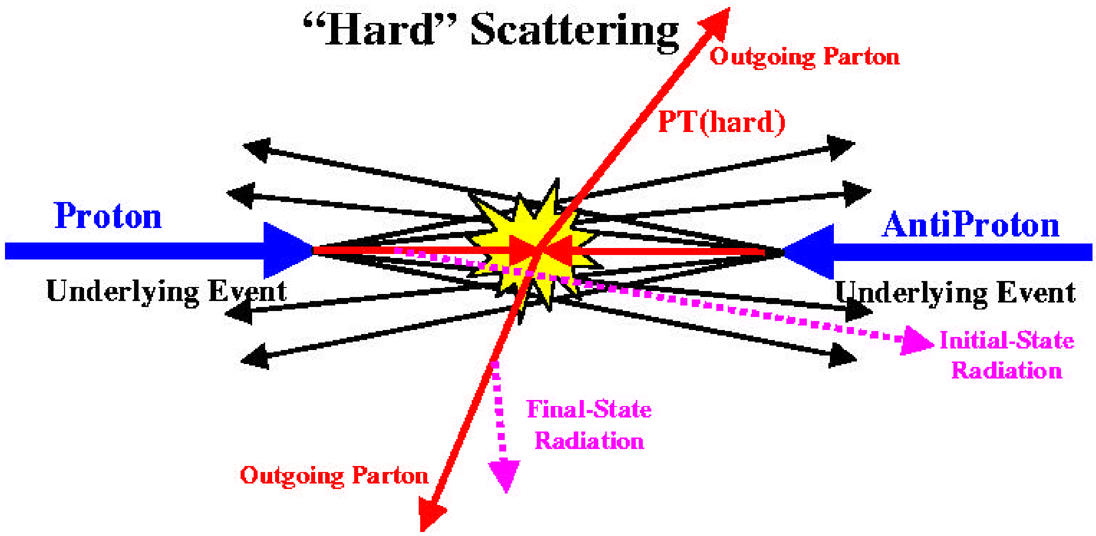

Figure 4: Proton – antiproton collision in which a "hard" 2-to-2 parton scattering with transverse momentum (P_T hard), has occurred. The resulting event contains particles that originate from the two outgoing partons and particles arising from the hadronization of the parent hadron remnants (the underlying event).

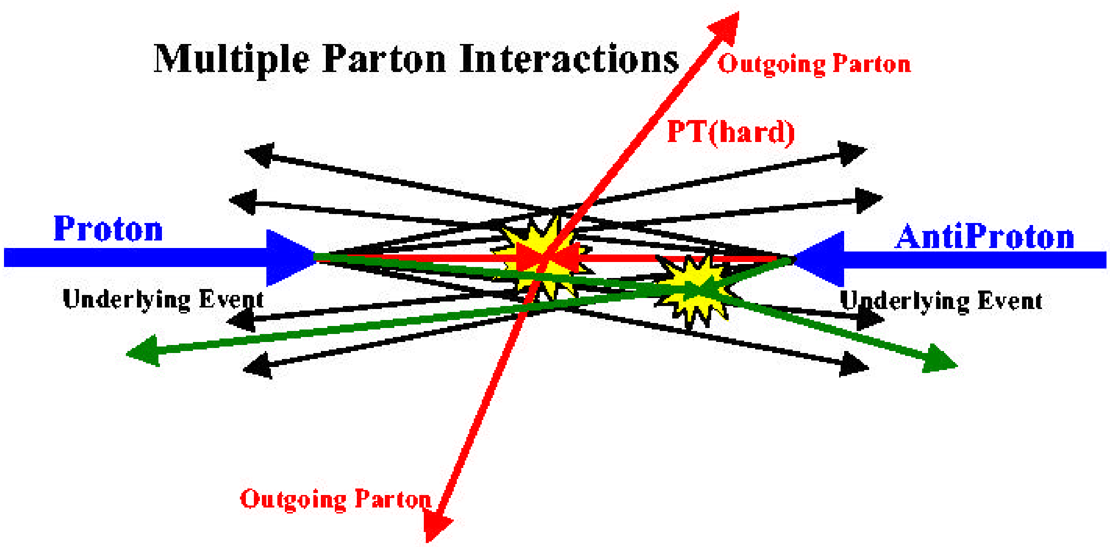

Figure 5: Proton-antiproton collision in which a multiple parton interaction has occurred. There is additional soft or semi-hard parton-parton scattering that contributes to the underlying reconstructed jet energy.

E. Jet Corrections for the Underlying Event

The underlying event produces ambient background energy in the calorimeter that gets clustered into jets but is not associated with the hard-scattered partons. Therefore, in order for jet energy to be related to that of the initial parent pre-hadronization partons, they have to be corrected for the underlying event energy. An amount of energy associated with the underlying event must be ascertained using Monte Carlo studies and subtracted from each jet.

III. CDF Jet Reconstruction

In collider experiments, a calorimeter jet appears as an energy deposit shared among several calorimeter towers. Jet clustering algorithms such as K_TCLUS and JETCLU are then used to associate clusters of this tower energy into calorimeter jets. They each start with a list of calorimeter towers and group the energetic ones that are close to each other in ? x f space together. They then combine the energy of the towers in each group according to their geometric location to determine the energy and momentum of the associated jet. The kinematic properties of these jets (i.e. momentum and energy) are related to the properties of the corresponding energetic parent partons produced during the hard-scattering process. In other words, through jet algorithms, the partons in their final hadronic state can be seen.

A. The JETCLU Algorithm

The standard CDF jet-clustering algorithm (JETCLU) is an iterative cone algorithm that forms jets by associating calorimeter together towers whose centers lie within a circle of specific radius R = v((? ?)^2 + (? f)^2) of 0.4, 0.7, or 1.0 units in ?-f space where η is the pseudorapidity, and φ is the azimuthal angle (*cf* Appendix for details on CDF coordinates and definitions). It begins by creating a list of towers above a fixed E_T threshold (1.0 GeV), which serve as the seed towers for the jet finder. The seed towers are then sorted into descending E_T order. Starting with the highest ET seed tower that acts as the geometric center (or axis) for the first cone in η x φ space a precluster is formed by clumping together adjacent seed towers within a particular cone radius of R in ? x f space. Additional preclusters are constructed by repeating the process starting with the next unclustered or unused seed tower.

The energy-weighted centroid of the cone is then calculated using the energy and geometry information of the calorimeter towers included within the cone. This new point (or axis) in η x φ

space is then used as the center for a new trial cone. As this calculation is iterated, the cone center "flows" until a "stable" solution is found, i.e., until the centroid of the energy deposits within the cone is aligned with the geometric axis of the cone.

B. The K_T CLUS Algorithm

Unlike the JETCLU clustering algorithm, the K_TCLUS algorithm successively merges pairs of nearby energetic calorimeter cell towers in order of increasing relative transverse energy. It contains a parameter D that controls termination of merging and a cone radius R, which characterizes the approximate size of the resulting jets, as shown in Figure 6.

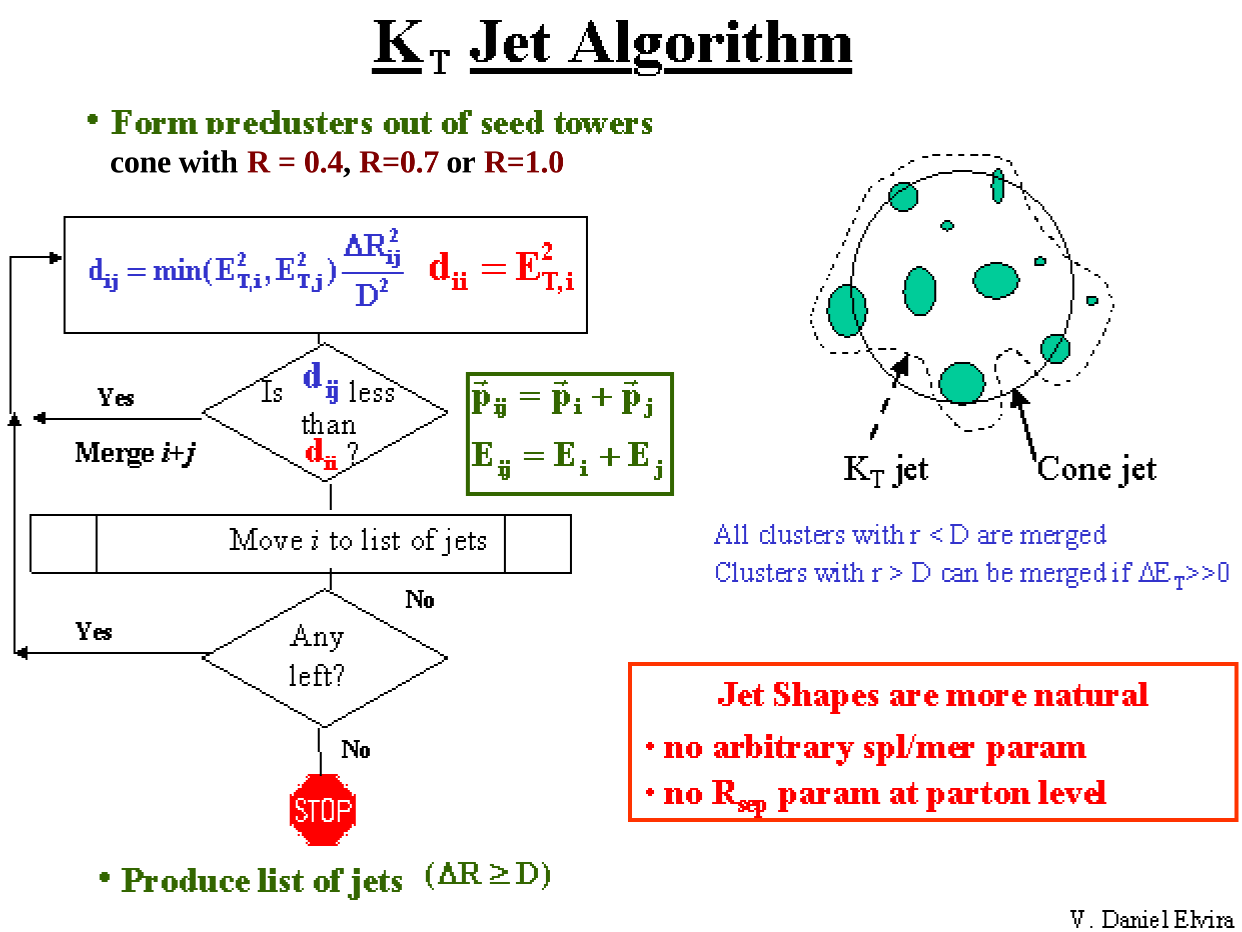

Figure 6: The K_TCLUS algorithm

C. Investigation Technique

To understand the effect of underlying event activity on the characteristics of the measured leading jets, we employed a powerful technique according to which we deliberately tweaked the calorimeter tower energy (according to equation 1) to simulate additional underlying event activity before jet reconstruction. The value xv (GeV) used to determine the amount by which the tower energy was tweaked was obtained from the non-negative randomly generated numbers that would normally constitute a Gaussian distribution with a mean of 0.2 and a width of 0.05. To simulate the effect of the underlying event on the transverse energy, E_T, in particular, each energy additive, xv (GeV) was divided by $\sin(?_{Tower})$. To test the sensitivity of each algorithm to the underlying event, we compared the jet distributions generated with altered to those produced by the straight data as reconstructed by each algorithm at the three different size parameters (R = 0.4, R = 0.7 and R = 1.0 units in η x φ space). The fractional difference in the E_T of the leading jet in each event before and after the tweaking was determined using equation 2. This is because the distribution of E_T should be flat as a function of η.

$$E_{\mathrm{Had}\,Tower(Altered)} = E_{HadTower(original)} + xv/[\,2*\sin(?_{\mathrm{HadTower}})] \quad (1)$$
$$E_{\mathrm{Em}\,Tower(Altered)} = E_{EmcTower(original)} + xv/[\,2*\sin(?_{\mathrm{EmTower}})]$$

$$\Delta E_{T(Fractional)} = \frac{[\,E_T Jet_{1\ (\mathrm{Tweaked})} - E_T Jet_{1\ (\mathrm{Straight})}]}{E_T\ Jet_{1\ (\mathrm{Straight})}} \quad (2)$$

The model used to derive the energy additive component, xv (GeV), was only a “simple model” speculation and did not possess all the properties of the underlying event. Nevertheless, this technique of investigating the underlying event by tweaking the tower energy before jet reconstruction is a very promising jet analysis tool. Better thought-out ways of altering the tower information prior to jet reconstruction in the future will give scientists invaluable insight into the effect of the underlying event on jet purity.

IV. Results and Discussion

The CDF detector collected the data sample of 4000 events used in this analysis on proton-antiproton collisions with a center of mass energy of 1.8 TeV. We generated jet distributions reconstructed (using the JETCLU and K_TCLUS algorithms at the three cone radii; R = 0.4, R = 0.7 and R = 1.0) from the straight data, then from the modified data. Figure 7 depicts the mean fractional difference in the E_T of the two leading jets, separately, in each event according to the results in table 2 as defined in equation 2.

Table of Results:

	K_TCLUS				JETCLU		
	Jet 1		Jet 2		Jet 1		Jet 2
R	**Mean** $\Delta E_{T(Fractional)}$	**Rms**	**Mean** $\Delta E_{T(Fractional)}$	**Rms**	**Mean** $\Delta E_{T(Fractional)}$	**Rms**	**Mean** $\Delta E_{T(Fractional)}$
0.4	0.1317	0.4323	0.2803	0.9232	0.1539	0.5068	0.2763
0.7	0.1649	0.2468	0.2745	0.4783	0.1789	0.2764	0.2823
1.0	0.2213	0.302	0.3499	0.5219	0.2487	0.3355	0.3575

Table 2: Mean fractional difference in the E_T of the leading jet in each event as described in equation 2.

We see that the mean fractional change in the E_T of the leading jets due to the simulated underlying event activity is systematically smaller in the K_TCLUS jets than in the JETCLU jets. In other words, the K_TCLUS generates purer (less sensitive the underlying event activity) jets than the JETCLUS algorithm. Also, smaller cone sizes resulted in purer leading jets.

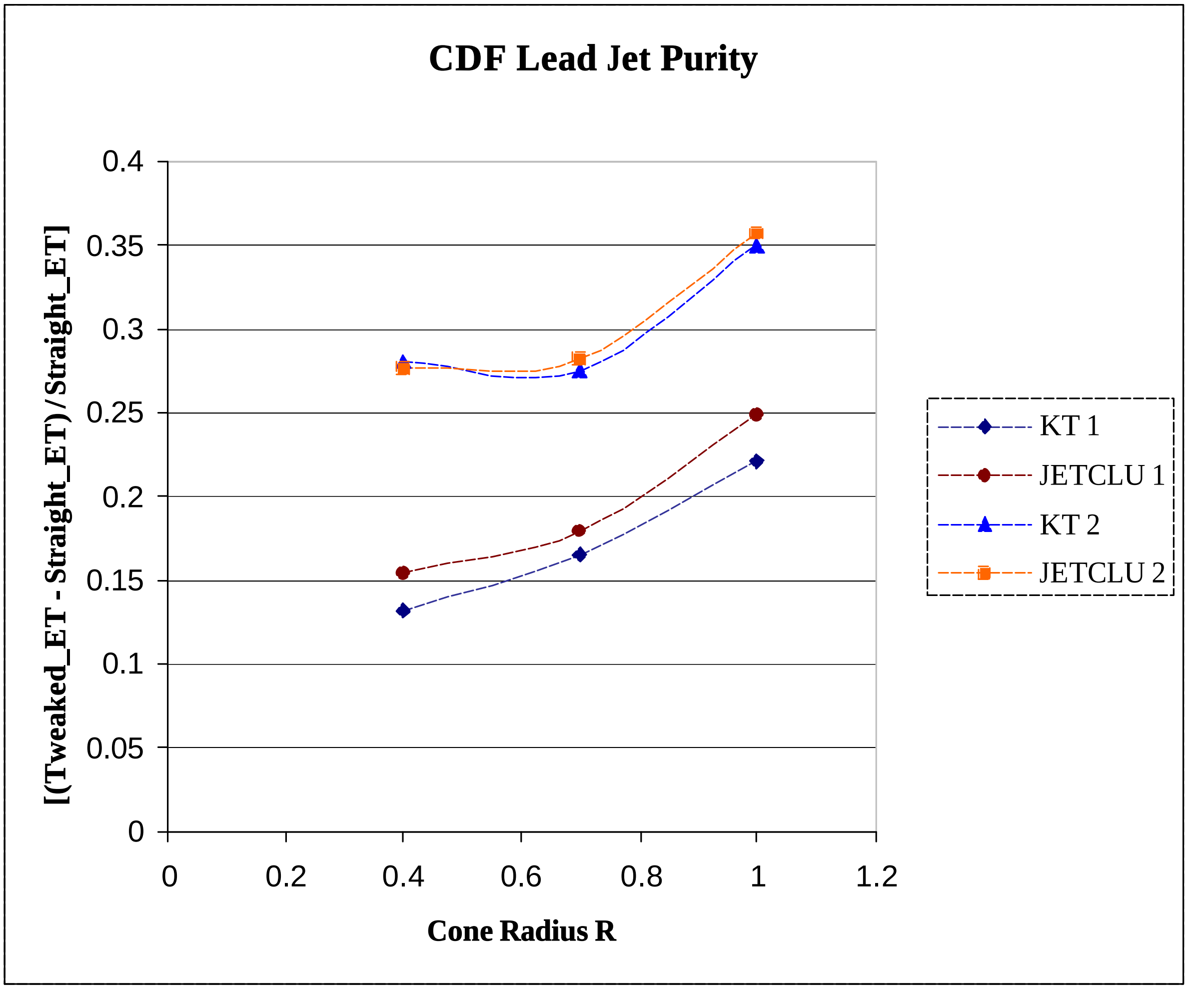

Figure 7: Fractional change in the E_T ($\Delta E_{T(Fractional)}$) of the two leading jets (separately) due to the simulated underlying event activity.

Figure 8 is a result sample of leading jet E_T distributions reconstructed using the JETCLU algorithm with a cone radius of 0.7 units in η x φ space. The distributions for the straight and altered data are superimposed in plot 3. Plot 4 is a distribution of the fractional change in the leading E_T jet, ($\Delta E_{T(Fractional)}$), of each event between the straight and altered data. From plot 3, we see that underlying event activity results in a systematic shift of the lead jet E_T distribution to the right. Plot 4 confirms the suspicion that underlying event activity generally adds extra energy to the measured jets. Nevertheless, we also see that it is not unusual for underlying event activity to result in lower E_T leading jets (negative values of the fraction $\Delta E_{T(Fractional)}$). A jet being split into two or more less energetic jets by underlying event activity could explain this as illustrated by the event in Figure 10, hence losing its hierarchy in the jet E_T ranks of the event.

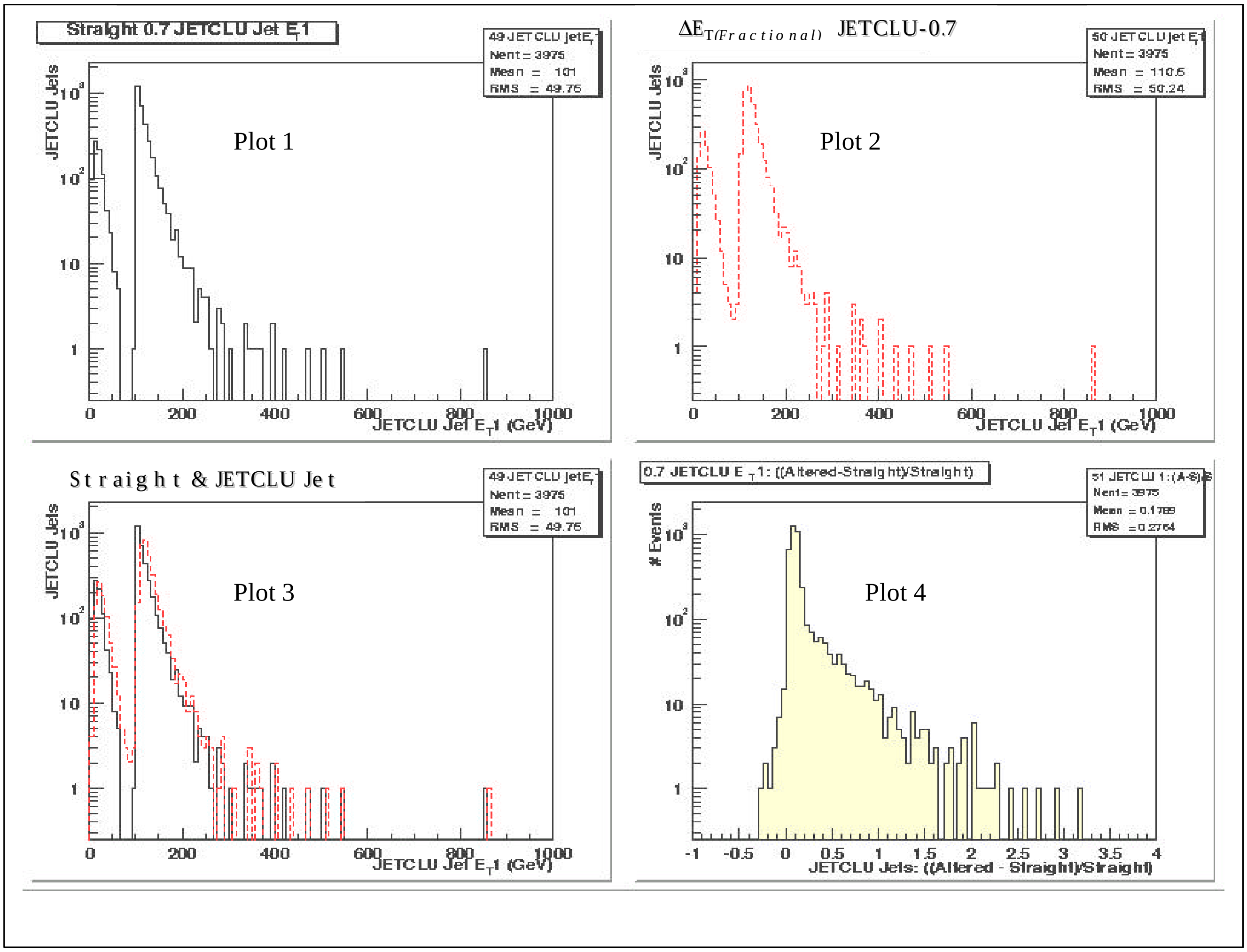

Figure 8: Lead jet E_T distribution histograms generated using the JETCLU-0.7 algorithm.

We also generated distributions showing the η x φ space separation between the leading jets in each event before and after we the additional, simulated underlying event activity (see Figure 9). Here, we see that underlying event activity may also change the position of the lead jets in η x φ space and reshuffle the E_T rankings among the jets in an event.

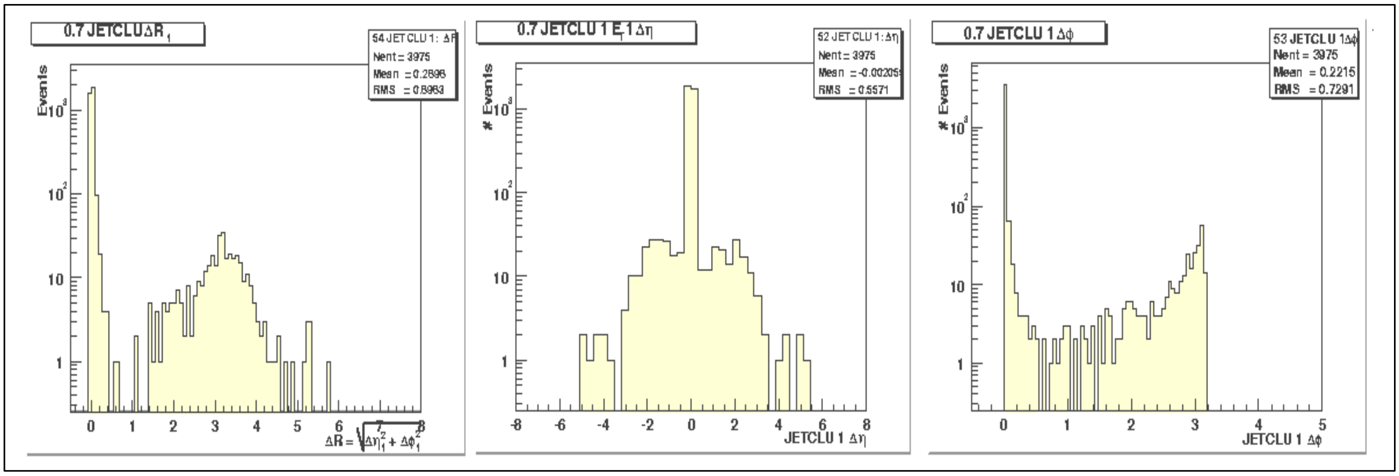

Figure 9: h x j space separation between the leading jets in each event before and after we the additional, simulated underlying event activity.

By zooming in on a particular event before and after tweaking the energy in the calorimeter tower cells, we are able to see how each jet algorithm reacted to the additional, simulated underlying event activity. First, we created a three-dimensional lego plot of the E_T in the 1,536 CDF calorimeter tower cells in the event. We then generated similar lego plots of the reconstructed jets. See Figure 10.

First of all, wee see the two energetic calorimeter jets, balanced in η x φ space, corresponding to the energetic partons that were produced during the hard-scattering process.

Before the tweaking, the JETCLU algorithm reconstructed two energetic jets as shown in Figure 10. After the tweaking, the leading JETCLU jet has been replaced by two less energetic jets. In other words, the jet E_T rankings in this event have been reshuffled by the underlying event activity that we simulated.

On the other hand, the E_T hierarchy of the jets reconstructed by the K_TCLUS algorithm has not been affected by the simulated underlying event activity. In this event, we see that the K_TCLUS algorithm was less sensitive to the simulated underlying event activity.

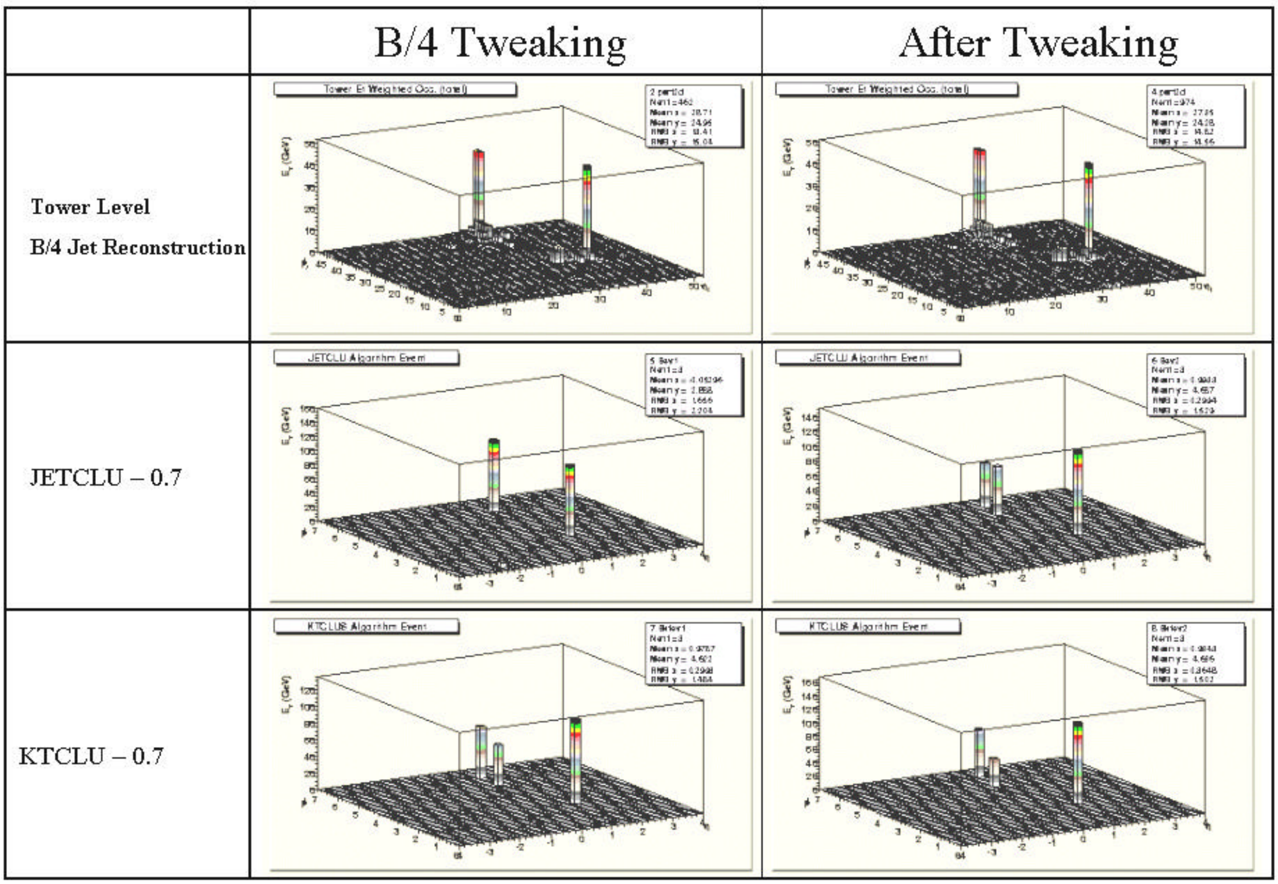

Figure 10: Reaction of the JETCLU and K_TCLUS algorithms to the additional, simulated underlying event activity.

V. CONCLUSION

In this analysis, our aim was to explore the sensitivity of the JETCLUS and K_TCLUS algorithms to the underlying event by tweaking the calorimeter tower energy (simulating additional underlying event activity) before jet reconstruction of CDF data in proton-antiproton collisions at a center of mass energy of 1.8 TeV. Our model favored the K_TCLUS over the JETCLU as the less sensitive algorithm to the underlying event activity. Smaller cone sizes generally resulted in purer (smaller fraction of the jet transverse energy, E_T, arising from the underlying event) leading jets. It was also ascertained that activity due to underlying event results in a systematic upward shift of the jet cross-section. It may also change the position of a jet in $\eta \times \varphi$ space and reshuffle the E_T rankings among the jets in an event.

Acknowledgements

I would like to thank the Summer Internships in Science and Technology committee giving me this research opportunity. Special thanks to my project supervisor and alternate supervisor, Dr. Jay R. Dittmann and Dr. Pierre Savard, respectively. Thank you for your invaluable advice, patience, guidance, and all the useful things you taught me about Quantum Chromodynamics and the CDF Run2 offline system. I would also like to thank Dr. Frank Chlebana and Sarah Demers for teaching me how to program in Root and AC++.

References

[1] T. Affolder *et al*. Charged Jet Evolution and the Underlyine Event in Proton-Antiproton Collisions at 1.8 TeV.

[2] R. Field, R. Haas and H. Frisch. The Underlying Event: Dijet vs. Z-jet.

[3] J. Dittmann. Measurement of the W + ≥ 1 Jet Cross Section in Proton-Antiproton Collisions at $\sqrt{s}$ = 1.8 TeV.

Appendix:

Coordinate Systems, Units and Variables for HEP

The CDF collaboration employs four primary right-handed coordinate systems: Cartesian (x,y,z), Cylindrical (r, φ, z), spherical (r, φ, θ) and a modified spherical system using transverse energy, pseudorapidity, and the azimuth (E_T, η, φ). The fourth coordinate system defines direction and magnitude rather than the three dimensional position. The positive z-axis lies along the beamline in the proton direction (east), the y-axis points vertically upward, and the positive x-axis points radially outwards in the horizontal plane of the Tevatron. The origin of the coordinate system is at the center of the detector. The azimuthal angle (φ) is measured clockwise from the positive x-axis. The polar angle (θ) is measured counterclockwise from the positive z-axis.

Variables of Collider Physics at CDF

The transverse component of the energy of a particle or group of particles, E_T, is defined as its total energy orthorgonal to the beam direction. In other words,

$$E_T = E\sin\theta \tag{3}$$

Because the initial particles in the beam have negligible transverse momentum components, by conservation of momentum, the vector E_T sum of all the resultant objects in an event must be zero.

The rapidity is a variable frequently used to describe the behaviour of particles in inclusively measured reactions. It is defined by:

$$y \equiv \frac{1}{2}\ln\left(\frac{E + p_\|}{E - p_\|}\right) \tag{4}$$

where E and $P_{\parallel}$ indicate the total energy and longitudinal momentum respectively. While rapidity is not Lorentz invariant, the first derivative is; thus the shape of a rapidity distribution will not change with boost in the longitudinal direction. In the limit that P >> m, the rapidity may be replaced

by the pseudorapidity η in terms of $\cos\theta = (E_Z/E)$ to yield equation 5.

$$\eta = -\ln\left(\frac{p+p_{\parallel}}{p-p_{\parallel}}\right)$$
$$\eta = -\ln\left(\tan\frac{\theta}{2}\right) \tag{5}$$

Figure 11 is an illustration of η x φ space.

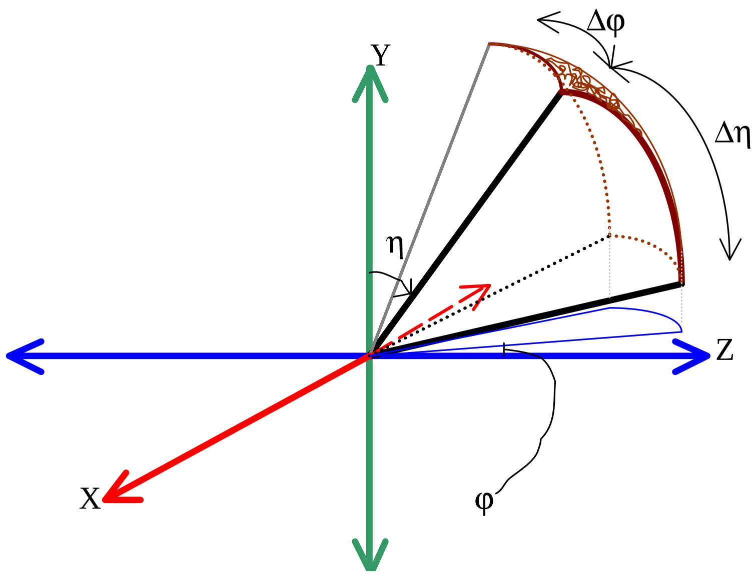

Figure 11: Graphical illustration of h x j space

Natural Units

As a standard of high-energy physics, all quantities are scaled by the two fundamental constants of relativistic quantum mechanics: Planck's constant

$$\hbar = \frac{h}{2\pi} = 1.055 \text{ X } 10\text{-}34 \text{ J}\bullet\text{sec} \tag{6}$$

and the speed of light in vacuum

$$c = 2.998 \text{ X } 10^{8} \text{ m sec}^{-1} \tag{7}$$

With the selection of units such that these quantities become dimensionless (i.e. $\hbar = c \equiv 1$), all quantities can easily be expressed in terms of energy, typically electron volts. It also follows that mass (m), momentum (mc), and energy (mc^2) all have the same units (GeV), as shown in the table below.

QUANTITY	UNITS
mass (m), momentum (mc), and energy (mc^2)	GeV
Length ($\hbar$/mc), time ($\hbar/mc^2$)	GeV^{-1}
Charge $(\hbar c)^{1/2}$	(dimensionless)

Table 3: Quantity units in high energy physics

As an exception to the convention, cross sections are expressed in terms of barns, where

$$1 \text{ b} = 1 \text{ X } 10^{-28} \text{ m}^{2} \tag{7}$$